647510 **MSC2MatC201x** **Seat No :________**

M.Sc. (Maths) Semester - 2 (*CBCS*) Examination
May/June-2022 (NEW COURSE)
Algebra – 2 (CORE)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q.1 (A) Answer any two of the following questions. **[10]**

(1) Write an example of a division ring which is not a field.
(2) Let $F \subset E \subset K$ be fields such that $[E : F] < \infty$ and $[K : E] < \infty$. Then show that $[K : F] = [K : E][E : F]$.
(3) Define splitting field and show that splitting field of a polynomial is unique up to isomorphism.

Q.1 (B) Answer any one of the following questions. **[04]**

(1) Check whether $x^3 - 5x + 10 = 0 \in Q[x]$ is reducible over Q or not.
(2) Define normal extension of a field and check whether $\mathbb{R}$ is a normal extension of Q or not.

Q.2 (A) Answer any two of the following questions. **[10]**

(1) Let E be a finite separable extension of a field F. If F is the fixed field of $G(E/F)$, then show that E is a normal extension of F.
(2) Define fixed field of the group of automorphisms E_H and show that E_H is a subfield of E.
(3) Show that the Galois group of $x^3 - 2 \in Q[x]$ is the group of symmetries of a triangle.

Q.2 (B) Answer any one of the following questions. **[04]**

(1) State and prove fundamental theorem of algebra.
(2) In usual notations show that $G(Q(\sqrt[3]{2})/Q)$ is the trivial group.

Q.3 (A) Answer any two of the following questions. **[10]**

(1) Show that the set of all $m \times n$ matrices over a ring R is a module over R.
(2) Let M be an R module and $x \in M$. Construct the smallest R submodule of M containing x.
(3) Justify : If N_1 and N_2 are two R-submodules of an R-module M, then $N_1 \cup N_2$ is also an R-submodules of M.

Q.3 (B) Answer any one of the following questions. **[04]**

(1) Define cyclic module and write an example of it.
(2) Define submodule and direct sum of modules.

Q.4 (A) Answer any two of the following questions. **[10]**

(1) State and prove fundamental theorem of R-homomorphism.
(2) Show that the submodules of the quotient module M/N are of the form U/N, Where U is a submodule of M containing N.
(3) Let R be a ring with unity and M be an R module. Then Show that M is simple if and only if $M \neq \{0\}$ and M is generated by some $0 \neq x \in M$.

Q.4 (B) Answer any one of the following questions. **[04]**

(1) Let $f: M \to N$ be an R-homomorphism. Define $\ker f$ and show that it is an R-submodule of M.
(2) Define free module and write an example of a nonzero free module.

Q.5 (A) Answer any two of the following questions. **[10]**

(1) Show that the multiplicative group of nonzero elements of a finite field is cyclic.
(2) If $f(x) \in F[x]$ is irreducible over F, then show that all roots of $f(x)$ have the same multiplicity.
(3) Show that every finitely generated module is a homomorphic image of a finitely generated free module.

Q.5 (B) Answer any one of the following questions. **[04]**

(1) Find $\theta \neq \sqrt{3} + \sqrt{5}$ such that $Q(\sqrt{3}, \sqrt{5}) = Q(\theta)$.
(2) State and prove Schur's lemma.

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